QCD Phase Diagram

Fluctuations and the Critical Point

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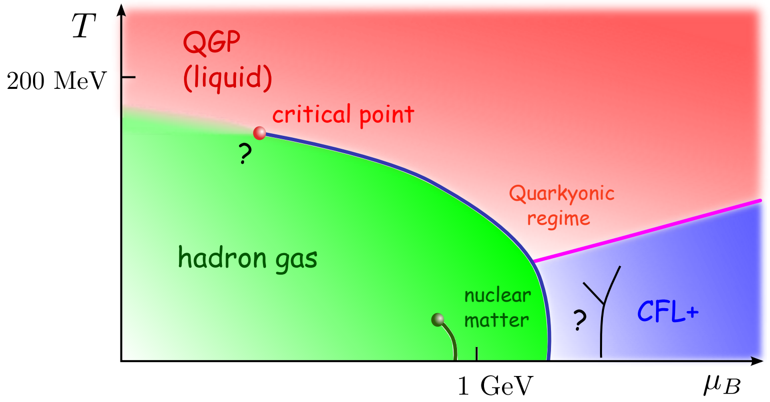
Outline


T
200 MeV
QGP
(liquid)
critical point
?
Quarkyonic
regime
hadron gas
nuclear
matter
?
CFL+
1 GeV
μ_B

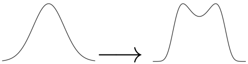
Critical point and fluctuations

The key equation:

$$P(x) \sim e^{S(x)} \qquad \textit{(Einstein 1910)}$$

At the critical point $S(x)$ has a “flat direction” or “soft-mode”.
Fluctuation measures diverge:

$$\langle x^2 \rangle = -\left(\frac{\partial^2 S}{\partial x^2} \right)^{-1} = VT\chi$$



In a grand canonical ensemble $P \sim e^{S-E/T-\mu N/T} = e^{pV/T}$

Fluctuations of order parameter and ξ

- Probability distribution for the order-parameter field:

$$P[\sigma] \sim \exp\{-\Omega[\sigma]/T\},$$

  Effective potential:

$$\Omega = \int d^3x \left[\frac{1}{2}(\boldsymbol{\nabla}\sigma)^2 + \frac{m_\sigma^2}{2}\sigma^2 + \frac{\lambda_3}{3}\sigma^3 + \frac{\lambda_4}{4}\sigma^4 + \ldots\right]. \quad \Rightarrow \quad \xi = m_\sigma^{-1}$$

- Moments of $\boldsymbol{q} = 0$ mode $\sigma_V \equiv \int d^3x\, \sigma(x)$:

  - Tree graphs. Each propagator gives ξ^2.
  - Scaling requires "running": $\lambda_3 \sim \xi^{-3/2}$, $\lambda_4 \sim \xi^{-1}$.

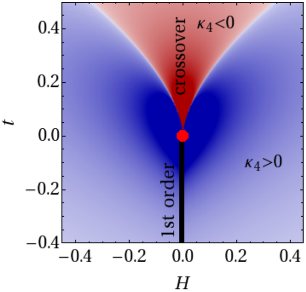
Sign

- The 2nd moment determined by ξ: $\langle \sigma_V^2 \rangle = VT\,\xi^2$.
- Higher moments also depend on the direction from the c.p.

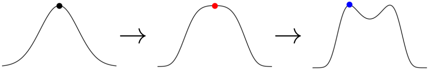
$$\langle \sigma_V^3 \rangle = 2VT^{3/2}\,\tilde\lambda_3\,\xi^{4.5}\,; \quad \langle \sigma_V^4 \rangle_c = 6VT^2\,[\,2(\tilde\lambda_3)^2 - \tilde\lambda_4\,]\,\xi^7\,.$$

E.g., on the crossover side: $\lambda_3 = 0$, i.e., $\langle \sigma_V^3 \rangle = 0$ and $\langle \sigma_V^4 \rangle_c < 0$.

- 2 relevant directions:



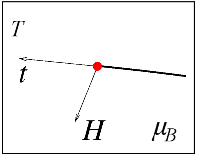
$P(\sigma_V)$ along $H = 0_\pm$



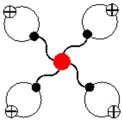
$\kappa_4 = 0$ $\quad$ $\kappa_4 < 0$ $\quad$ $\kappa_4 > 0$, $\langle \sigma_V \rangle \neq 0$

QCD

- In QCD $(t, H) \to (\mu - \mu_{\rm CP}, T - T_{\rm CP})$



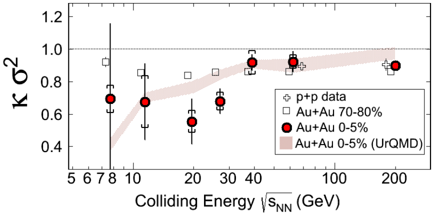
- $$\langle(\delta N)^4\rangle_c = \langle N\rangle + \langle\sigma_V^4\rangle_c \left(\frac{g}{T}\int_{\boldsymbol p} \frac{n_{\boldsymbol p}(1\pm n_{\boldsymbol p})}{\gamma_{\boldsymbol p}}\right)^4 + \ldots,$$

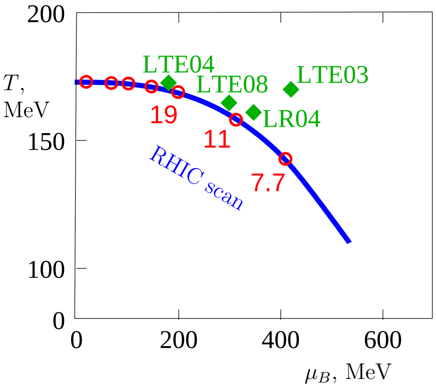


$\langle\sigma_V^4\rangle_c < 0$ means $\omega_4(N) \equiv \dfrac{\langle(\delta N)^4\rangle_c}{\langle N\rangle} < 1$

Estimates in Athanasiou-Rajagopal-MS 2010.

RHIC energy scan

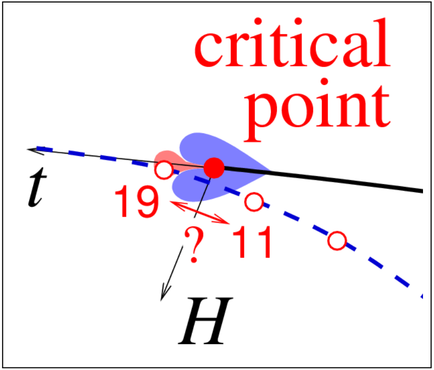


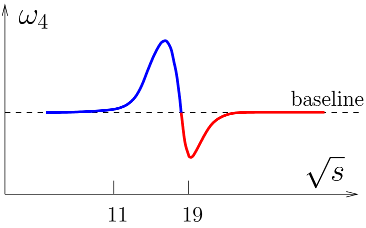


- Negative contribution to κ_4 around 19 GeV ($\mu_B \sim 200$ MeV).
- O(magnitude) consistent with estimates at 19 GeV.

Acceptance effects important (Asakawa-Kitazava 2012 Bzdak-Koch 2012) .

A scenario





- Assuming critical region $\Delta\mu_B \sim \mathcal{O}(100)$ MeV.

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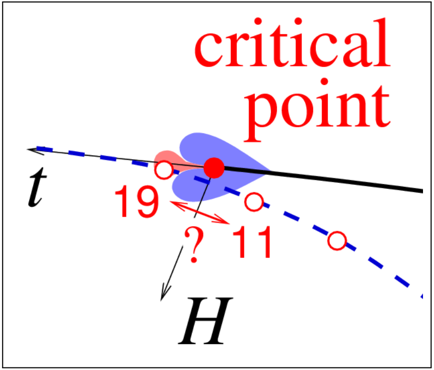
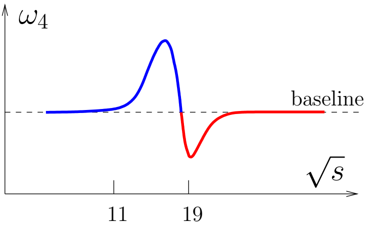
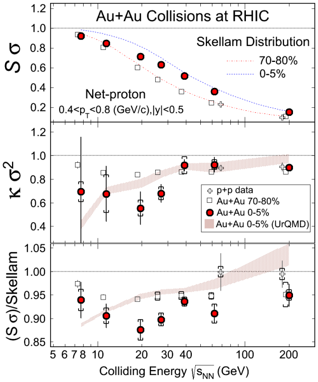




- Assuming critical region $\Delta\mu_B \sim \mathcal{O}(100)$ MeV.
- To confirm or rule out – need data at 15 GeV.
- First order transition signatures at 11 and 7.7 GeV?

Questions and Thoughts



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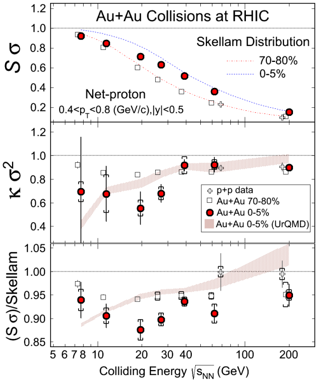

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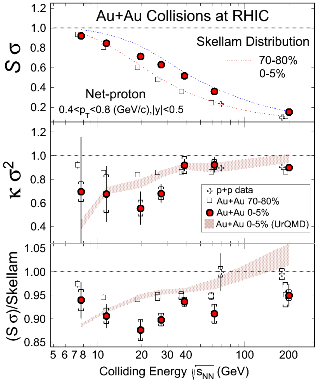

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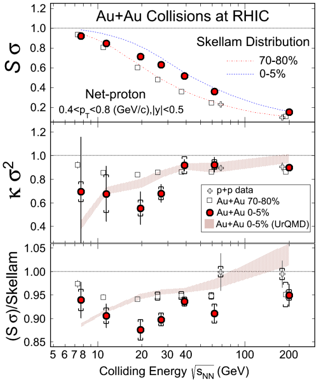

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- Why $S\sigma <$ Skellam?
  - $S\sigma = \dfrac{\kappa_3}{\kappa_2}$. $\kappa_2/$Skellam?
  - $S_{\rm critical} < 0$ above c.p. (Asakawa-Ejiri-Kitazawa 2009) - “memory”?

- Important to study dynamical evolution of fluctuations.

Summary I

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  15 GeV will be interesting.
- For theory: dynamics and fluctuations.

Dynamics of charge fluctuations

with Bo Ling, Todd Springer

Charge current fluctuations

Relativistic stochastic hydrodynamics (Kapusta-Müller-MS 2011)
for charge density, $n = J^\mu u_\mu$,

$$\nabla_\mu J^\mu = 0, \qquad J^\mu = n u^\mu + \Delta J^\mu + I^\mu.$$

$$\Delta J^\mu = \sigma T \Delta^{\mu\nu} \nabla_\nu \left(\frac{\mu}{T}\right) \qquad (\Delta^{\mu\nu} = -g^{\mu\nu} + u^\mu u^\nu)$$

$$\langle I^\mu(x) I^\nu(y) \rangle = 2\sigma T \Delta^{\mu\nu} \delta^{(4)}(x-y).$$

σ – conductivity. Related to diffusion coeff. $D = \dfrac{\sigma}{\chi}$, where $\chi = \dfrac{dn}{d\mu}$.

Cf. Kapusta-Torres-Rincon 2012: $+$ diffusion, $-$ mixing ($\mu_B = 0$).

Diffusion in Bjorken flow

We are solving

$$\nabla_\mu \left(n u^\mu + \Delta J^\mu + I^\mu \right) = 0.$$

in Bjorken coordinates:

$$ds^2 = -\underbrace{d\tau^2}_{dt^2} + \underbrace{\tau^2 d\xi^2}_{dl^2}$$

Diffusion in comoving frame: $dl^2 = D dt \quad \Rightarrow \quad \tau^2 d\xi^2 = D d\tau$.

$$(\Delta\xi_{\rm diff})^2 = \int_\tau^{\tau_{\rm f}} D \, \frac{d\tau}{\tau^2}.$$

Charge correlations and crossover

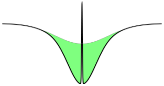
- $$\int d\xi\, e^{ik_\xi \xi} \langle \delta n(\xi) \delta n(0) \rangle = k_\xi^2 \int_{\tau_0}^{\tau_\mathrm{f}} \frac{2\sigma T}{\tau} e^{-2H} d\tau \qquad \Bigg(H = k_\xi^2 \underbrace{\int_\tau^{\tau_\mathrm{f}} D \frac{d\tau}{\tau^2}}_{(\Delta\xi_\mathrm{diff})^2} \Bigg)$$

  integrate by parts and use $\sigma/D = \chi$:

$$= \quad \Big[\underbrace{(\chi T \tau)_{\tau_\mathrm{f}}}_{\to \delta(\xi)} - \underbrace{\left(\chi T \tau e^{-2H}\right)_{\tau_0}}_{\text{a wide Gaussian}} \Big] \; - \; \underbrace{\int_{\tau_0}^{\tau_\mathrm{f}} \frac{d\,(\chi T \tau)}{d\tau} e^{-2H} d\tau}_{\text{superpos. of Gaussians}}$$

  Delta-function – trivial: $\langle \frac{dN}{dy_1} \frac{dN}{dy_2} \rangle = \frac{dN}{dy} \delta_{y_1, y_2}$.

  The Gaussians $\to$ two-particle correlations.



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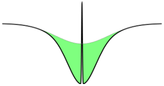
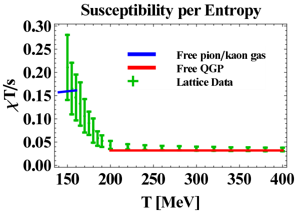


- Under Bjorken $\chi T\tau \sim \dfrac{\chi T}{s}$

  – constant in conformal plasma.

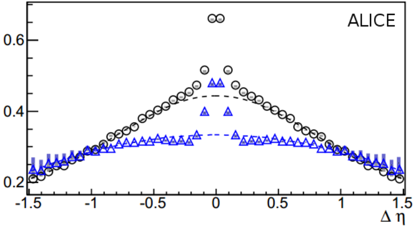
  Crossover contributes.

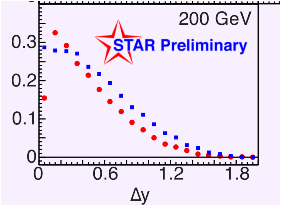


Comparison with experiment

Balance function $\sim$ difference of US-LS correlators:

$$B(y_1, y_2) \overset{\text{const}}{=} -\langle (N_+ - N_-)_1 (N_+ - N_-)_2 \rangle = 2(\langle N_+ N_- \rangle - \langle N_+ N_+ \rangle).$$





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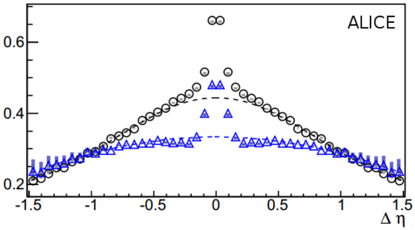
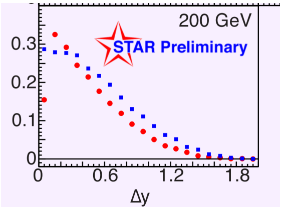
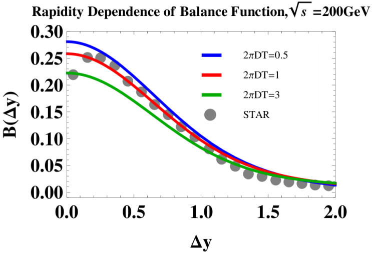




- Amplitude depends on EOS, $\chi T/s$.
- Width depends on diffusion coeff.

$$\hat{D} \equiv DT \sim \frac{1.}{2\pi}$$

Suggests D is small – characteristic of strongly coupled medium.



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- Future: stochastic hydro near the QCD critical point.