



# Bridging the gap between theoretical and experimental results on higher moments

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# The Plan

- ❖ Not actually going to bridge the gap
  - ❖ Just going to talk about it
- ❖ What does the data tell us so far?
- ❖ What progress have we made already?
- ❖ Where can we go from here?

# Why higher moments?

If we start with the log of the QCD partition function

$$\frac{p}{T^4} \equiv \frac{1}{VT^3} \ln[Z(V, T, \mu_B, \mu_S, \mu_Q)]$$

then take derivatives with respect to the chemical potential

$$\chi_n^x \equiv \left. \frac{\partial^n p/T^4}{\partial \hat{\mu}_x^n} \right|_{\mu=0}$$

we find that the cumulants of conserved quantities give us the susceptibilities

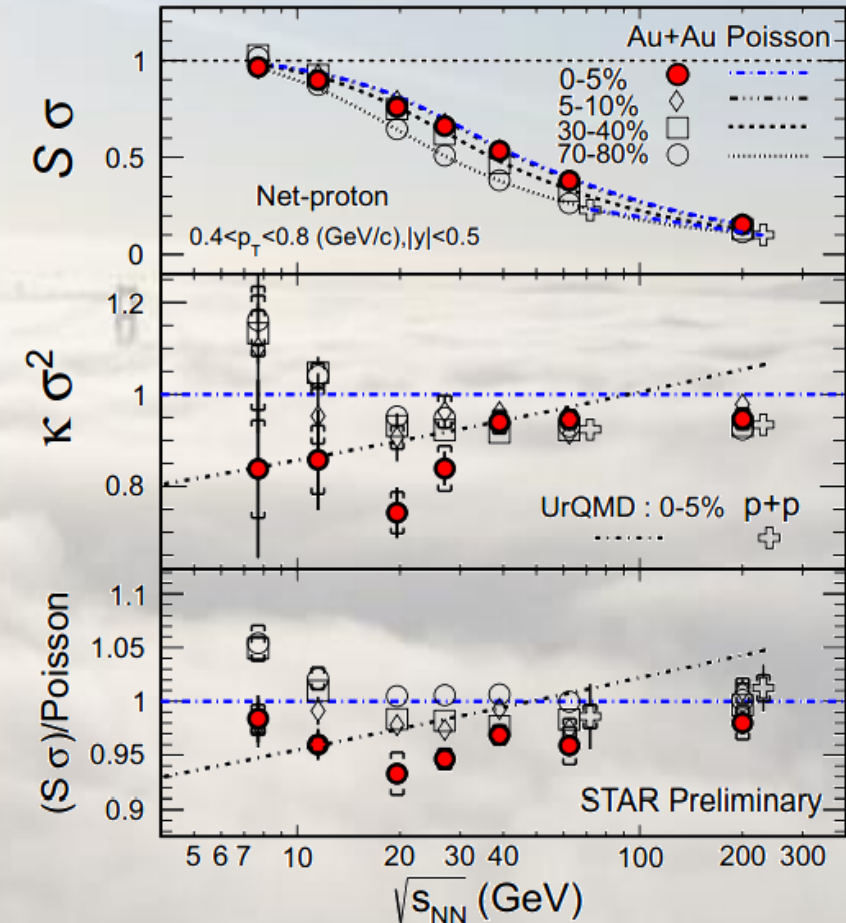
$$\chi_n^x = \frac{1}{VT^3} \langle (\delta N_x)^n \rangle = \frac{1}{VT^3} \kappa_n$$

**Critical point fluctuations would cause these to diverge.  
Values relate to locations on the phase diagram.**

# The Data (Net Proton)

- ❖ Very little energy/centrality dependence beyond the changing  $N_p/N_{\bar{p}}$  ratio
- ❖ Features are on order of 5-10%
- ❖ No compelling evidence for critical point fluctuations

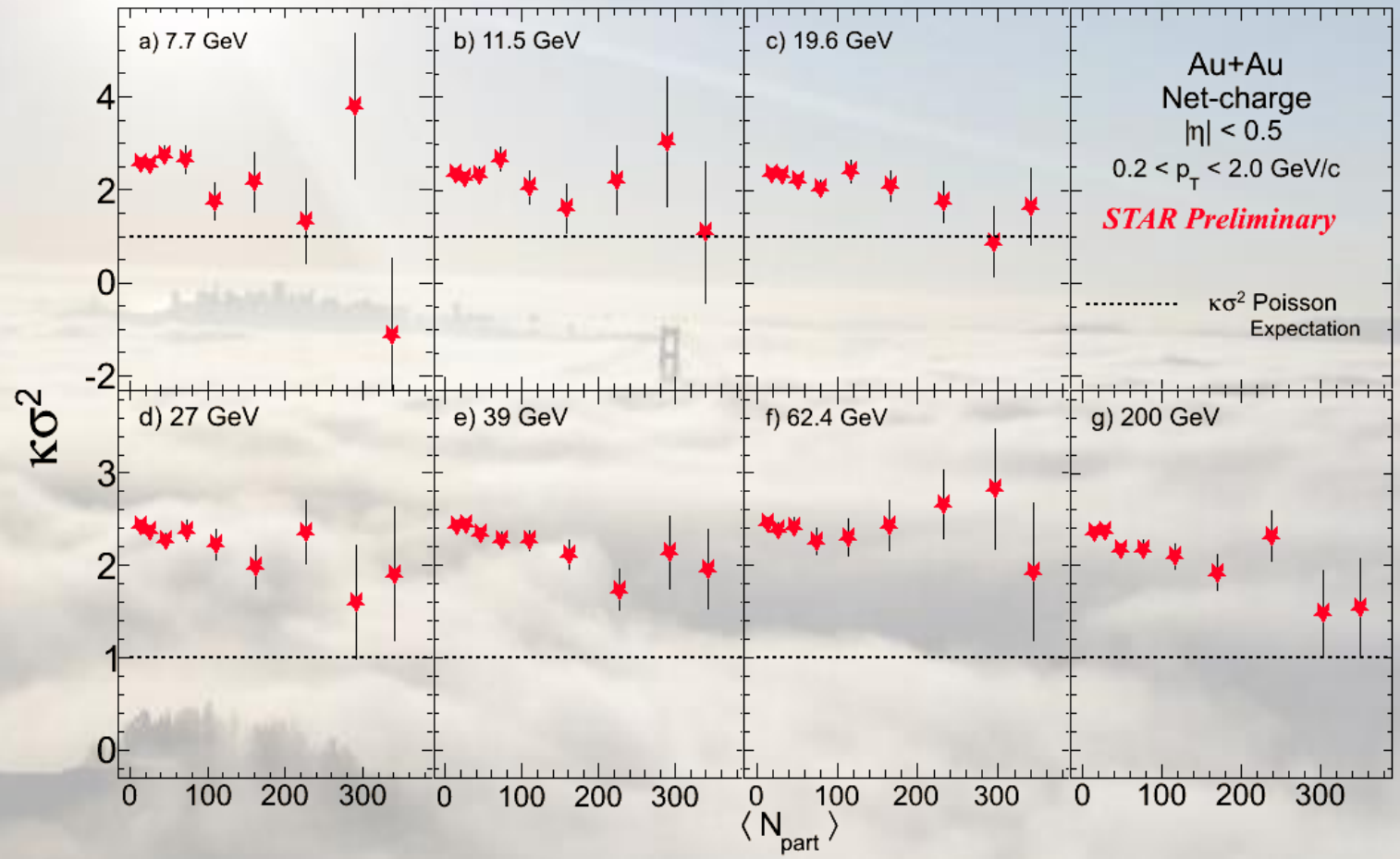
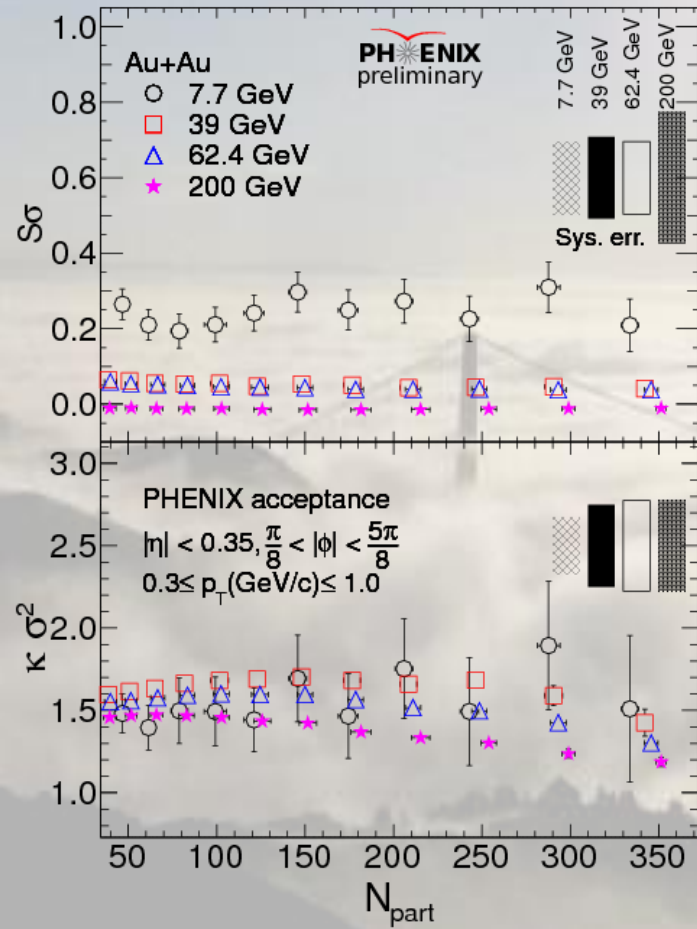
X. Lou for STAR, arXiv:1306.3106



# The Data (Net Charge)

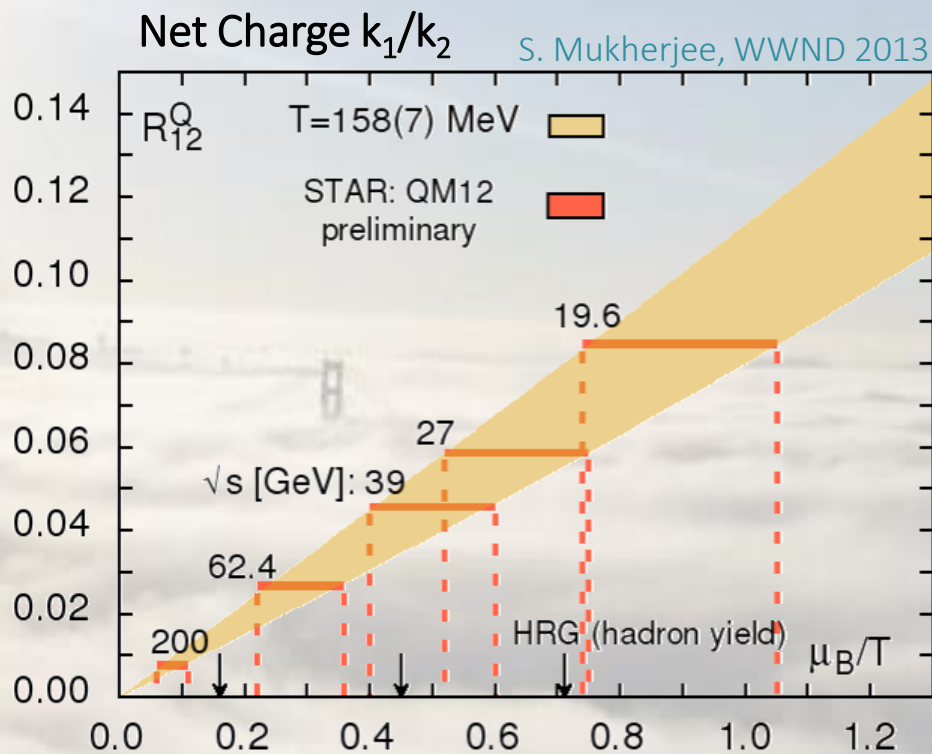
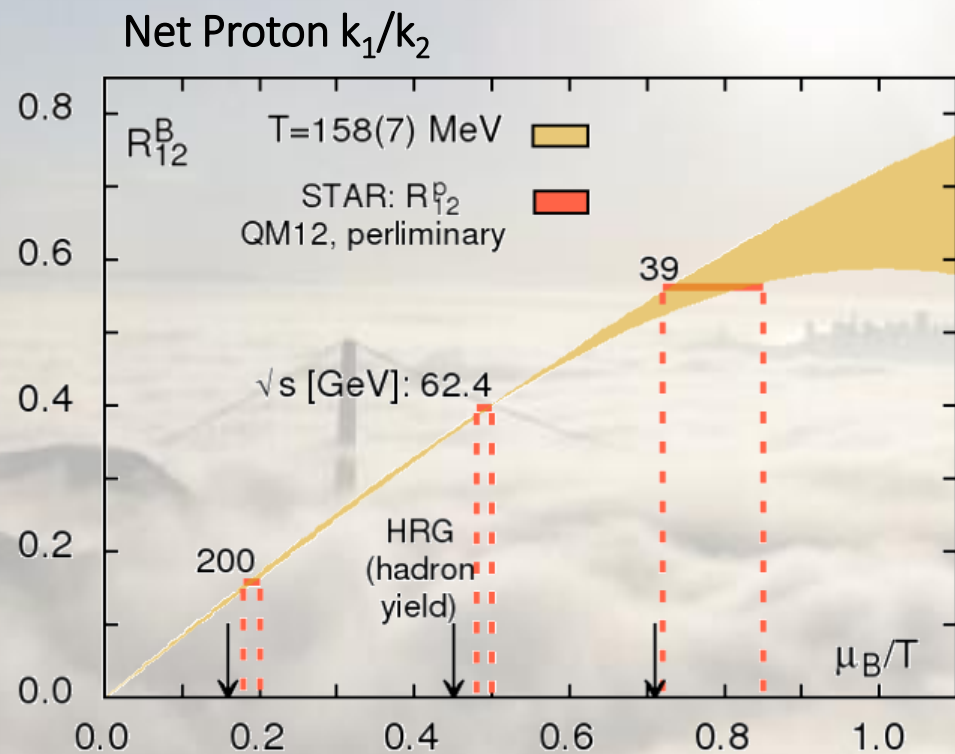
J. Mitchel, Quark Matter 2012

N. Sahoo, Quark Matter 2012



**PHENIX results are systematically lower than STAR's.**

# The Data (Coupled w/ Lattice)



**STAR Net Charge and Net Proton results  
give inconsistent values of  $\mu_B/T$ ?**

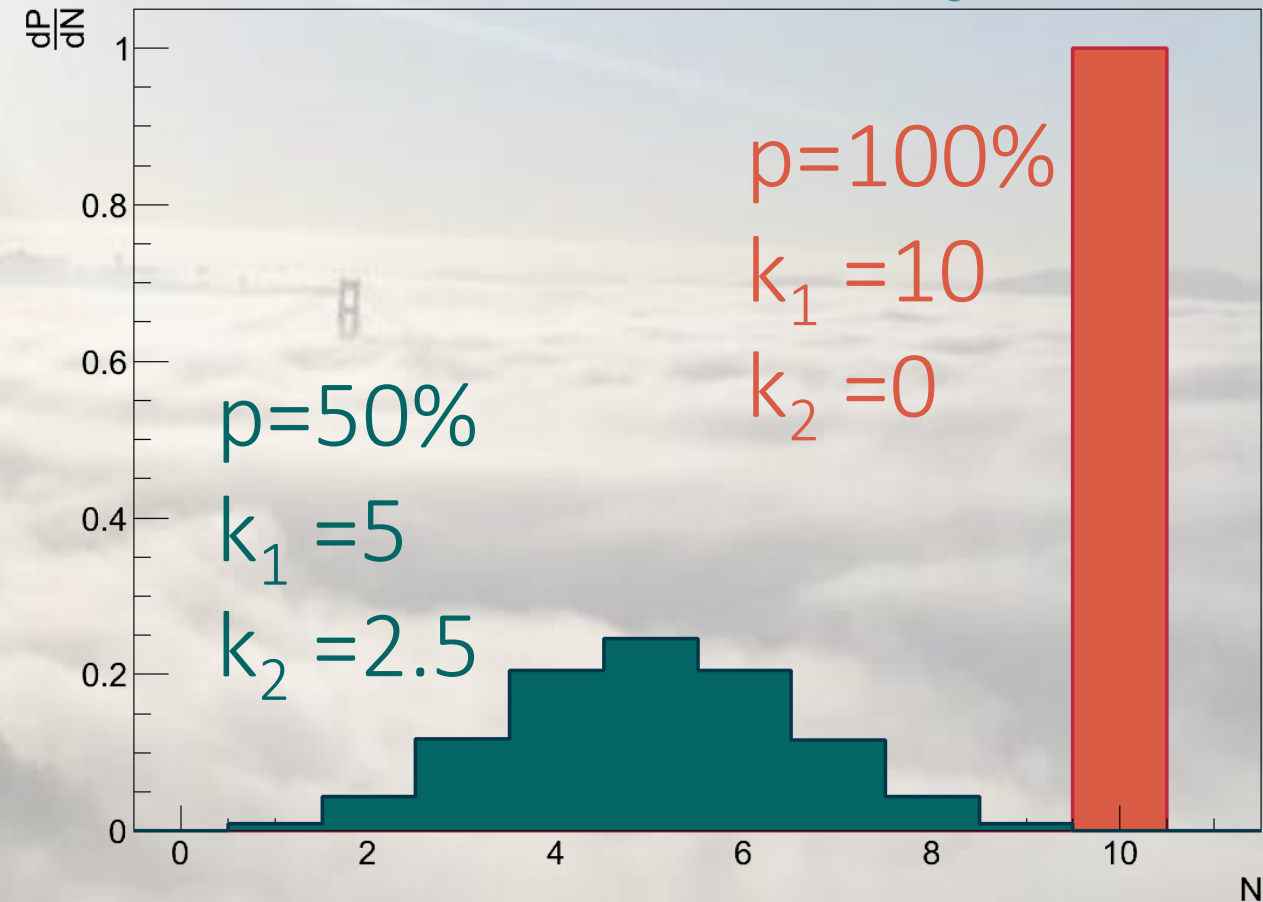
# What Does the Data Tell Us?

- ❖ We see no compelling evidence for a critical point
  - ❖ Little energy/centrality dependence of  $k_4/k_2$
- ❖ Possible systematic issues affecting STAR and PHENIX in different ways
- ❖ Current lattice results seem inconsistent with “lower” moments
- ❖ There is more work that needs to be done in order to understand these results...

# Efficiency Losses

- ❖ Push results toward Poissonian expectations
- ❖ Cause artificially small statistical and systematic errors
- ❖ Can be corrected analytically
  - ❖ Derivation:  
[http://nuclear.ucdavis.edu/~sangaline/research/papers/efficiency\\_corrections.pdf](http://nuclear.ucdavis.edu/~sangaline/research/papers/efficiency_corrections.pdf)
  - ❖ Generally accepted by the community
    - ❖ Will be included in the final STAR and ALICE results

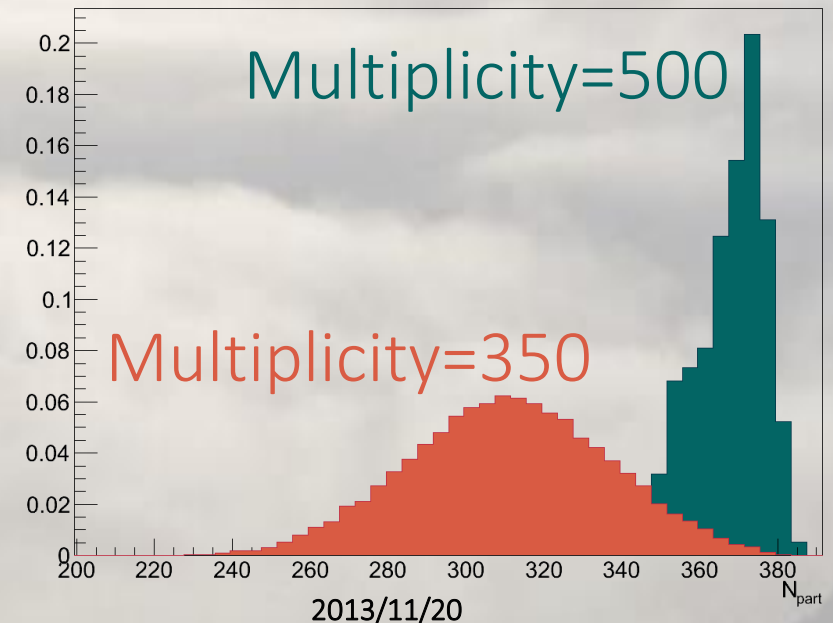
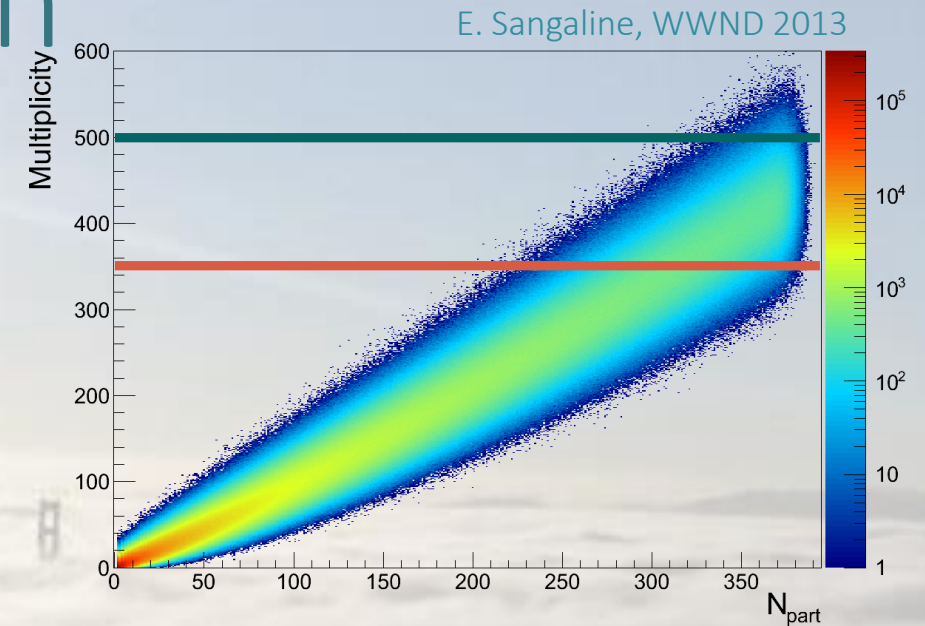
E. Sangaline, WWND 2013





# Finite Centrality Resolution

- ❖ Introduces centrality, collision energy, and experiment dependent biases
- ❖ Analytical corrections exist if the volume distribution is known
  - ❖ Model dependent (e.g. Glauber)
  - ❖ No plans for publication in either STAR or PHENIX
- ❖ Reduced by maximizing phase space used in centrality determination
  - ❖ Still a large effect, especially in net charge analyses

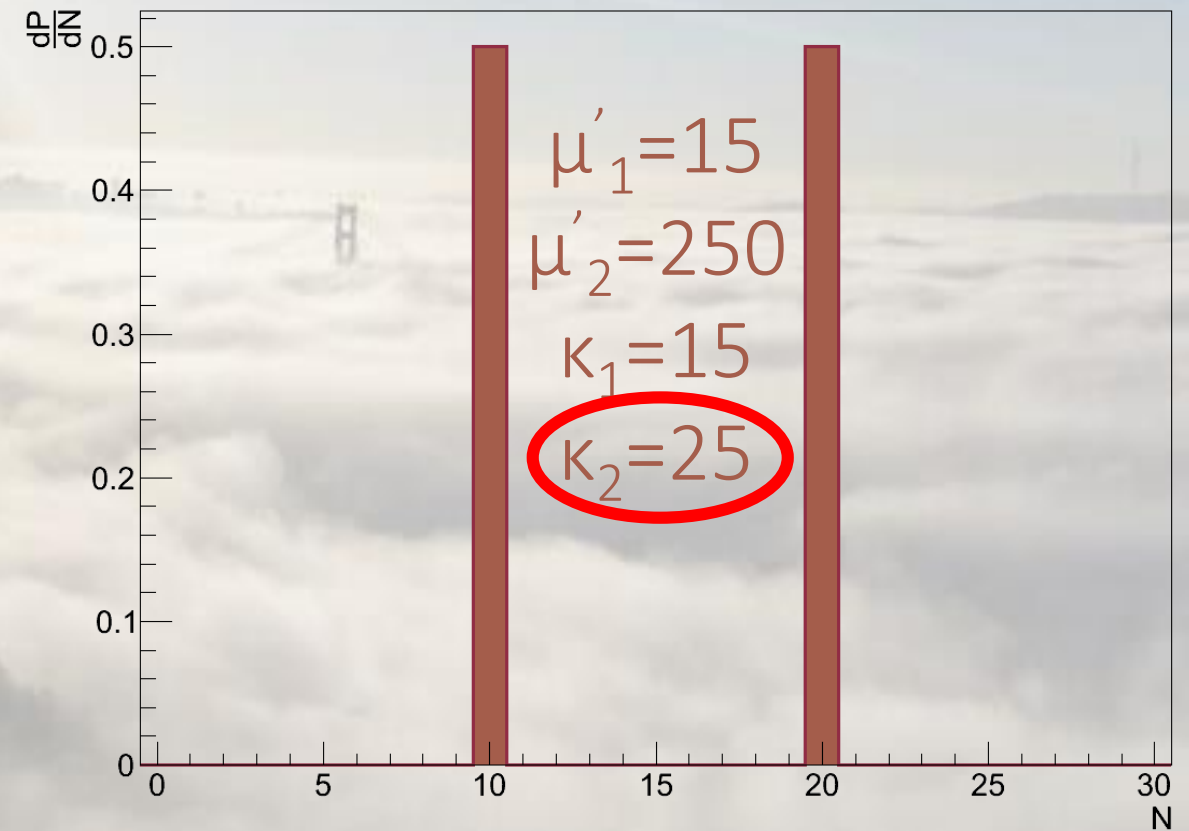
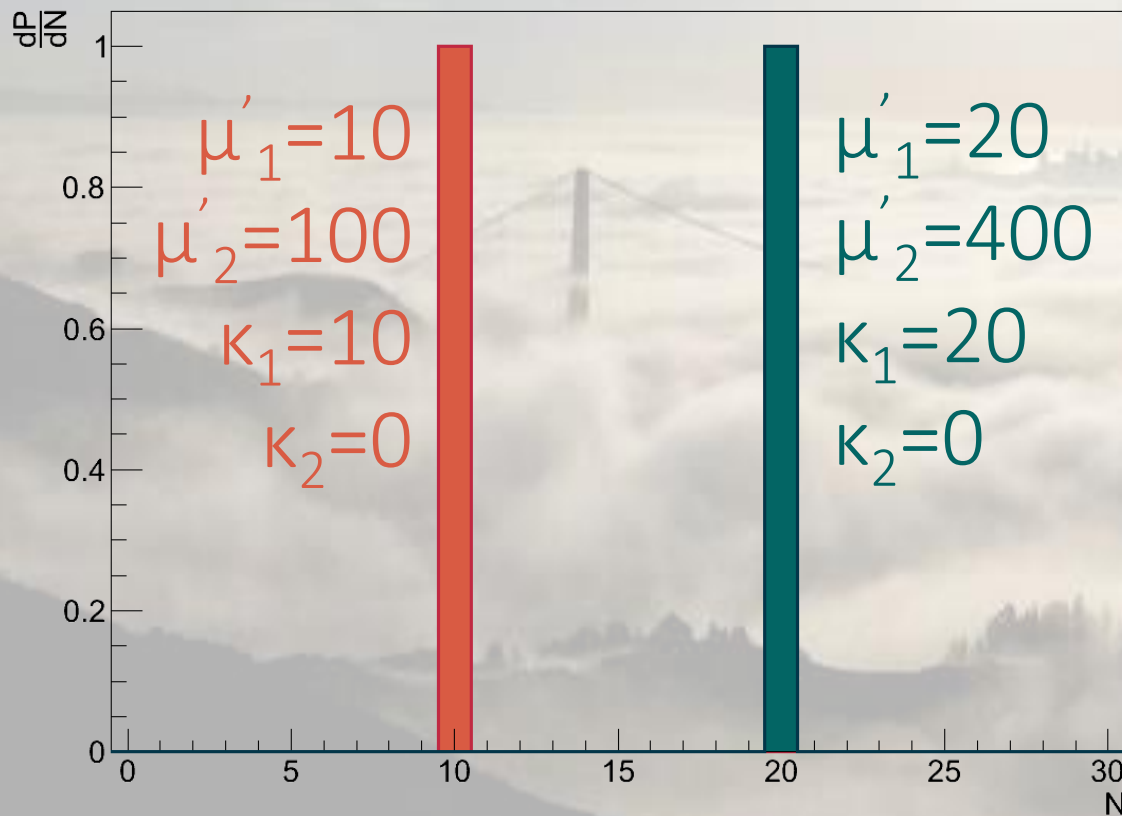


# Centrality Resolution

Multiple distributions with the same cumulant ratios



Convoluting distribution with different cumulant ratios

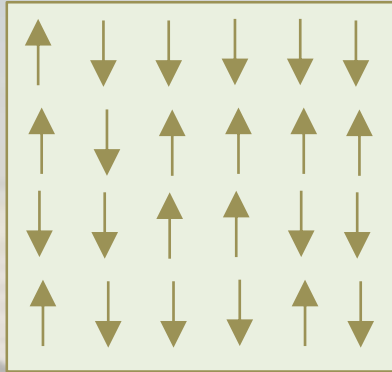


# Other Effects

- ❖ Acceptance
  - ❖ Conservation suppresses fluctuations for larger acceptances
  - ❖ Small acceptances exclude particles and have an efficiency-like effect
    - ❖ Depends on relationship between position and momentum space
  - ❖ Likely explanation of STAR/PHENIX net charge results
- ❖ Autocorrelation in centrality determination from resonance decays and jets
- ❖ Impact of jets with partial acceptance on charge fluctuations
- ❖ Time evolution of  $\mu_B$  and  $T$ 
  - ❖ How much does history of the collision affect measurements?
- ❖ Finite size effects
  - ❖ Limited volume
  - ❖ Limited equilibration

# The 2D Ising Model

A 2D Lattice of Dipole Spins



Hamiltonian Given By

$$H = \sum_{i,j} \sum_{k,l \in \text{neighbors}_{i,j}} -S_{ij}S_{jk}$$

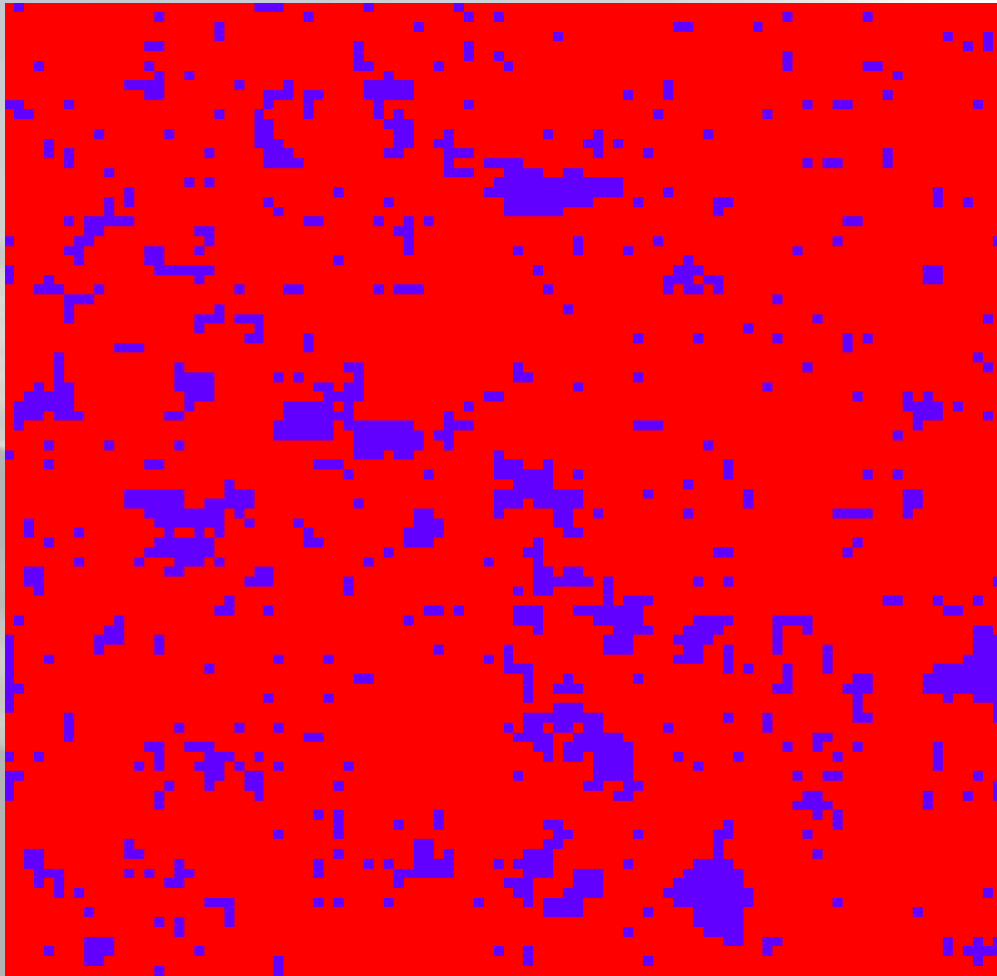
Magnetic Susceptibility Given By

$$\chi = \frac{\partial M}{\partial T} = \frac{\langle M^2 \rangle - \langle M \rangle^2}{T}$$

Diverges at  $T_c$  for an infinite lattice, analogous to the goal of higher moments analysis.

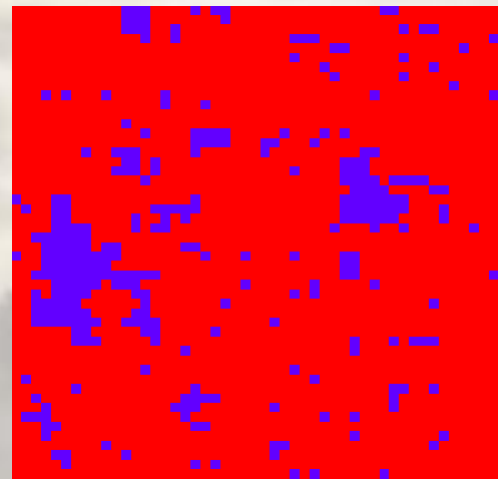
One of the simplest thermodynamic systems with a phase transition. Useful for a qualitative understanding of some effects.

# Ising Model Simulations

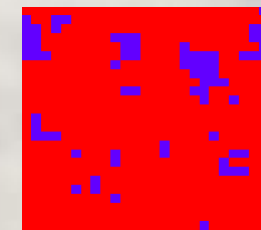


L=100

- ❖ Metropolis algorithm for sampling
- ❖ Various lattice sizes
- ❖ Non-periodic boundaries
- ❖ Absolute magnetization measured ( $|M|$ )



L=50



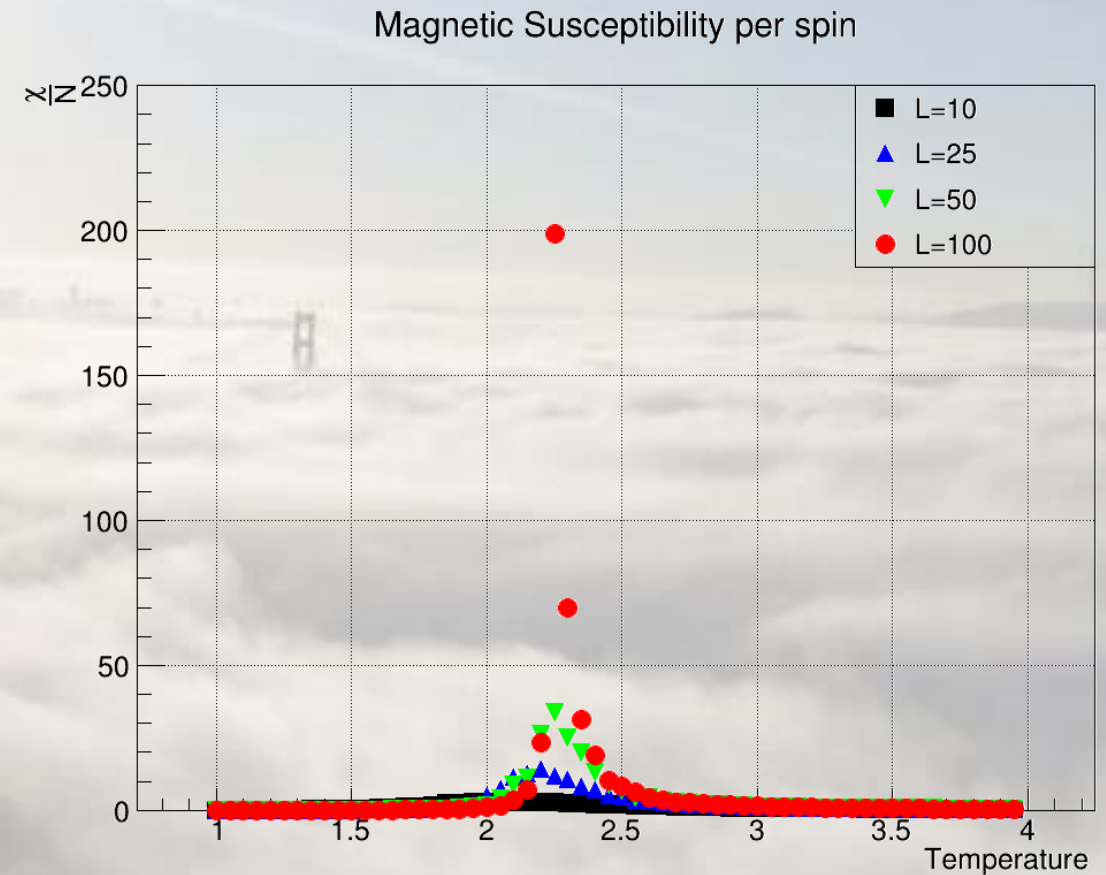
L=25



L=10

# Magnet Susceptibilities and Finite Size

- ❖ Divergence rapidly disappears for small system sizes
  - ❖ Larger effect for higher order cumulants
- ❖ We only measure a handful of protons on average
  - ❖ How much of a divergence should we expect if there is a critical point?
  - ❖ And if the system isn't equilibrated at a large scale?



# Conclusions

- ❖ We have a wide range of results from RHIC
  - ❖ Possible features are very subtle
- ❖ Experimental effects need to be eliminated as far as possible and expectations need to be made more realistic
  - ❖ A lot of progress has been made
  - ❖ A lot more needs to be made